14/8/2020

B.Sc. Part I <u>TRIGONOMETRY</u> Ist Paper

Summation of series (contd.)

<u>1.</u> Sum the series

$$e^{\alpha}.\cos\beta - \frac{1}{3}e^{3\alpha}\cos 3\beta + \frac{1}{5}e^{5\alpha}\cos 5\beta \text{---} \text{to}\infty$$

<u>Soln</u> Let the given series be denoted by C.

$$\Rightarrow C = e^{\alpha}\cos\beta - \frac{1}{3}e^{3\alpha}\cos 3\beta + \frac{1}{5}e^{5\alpha}\cos 5\beta \text{----to}\infty$$

Let $$S = e^{\alpha}\sin\beta - \frac{1}{3}e^{3\alpha}\sin 3\beta + \frac{1}{5}e^{5\alpha}\sin 5\beta \text{-- to}\infty$$

$$\Rightarrow C+iS = e^{\alpha}(\cos\beta + i\sin\beta) - \frac{1}{3}e^{3\alpha}(\cos 3\beta + i\sin 3\beta)$$
$$+ \frac{1}{5}e^{5\alpha}(\cos 5\beta + i\sin 5\beta) \text{----to}\infty$$

$$\Rightarrow C+iS = e^{\alpha}.e^{i\beta} - \frac{1}{3}e^{3\alpha}.e^{3i\beta} + \frac{1}{5}.e^{5\alpha}.e^{5i\beta} \text{---to}\infty$$

$$\Rightarrow C+iS = e^{\alpha+i\beta} - \frac{1}{3}e^{3(\alpha+i\beta)} + \frac{1}{5}e^{5(\alpha+i\beta)} \text{---to}\infty$$

Put $e^{\alpha+i\beta} = x$

$$\Rightarrow C+iS = x - \frac{x^3}{3} + \frac{x^5}{5} \text{----to}\infty$$

$$\Rightarrow C+iS = \tan^{-1}x$$

$\Rightarrow C + iS = \tan^{-1}\left(e^{\alpha + i\beta}\right)$ —— (1)

Replacing i by $-i$, we get

$C - iS = \tan^{-1}\left(e^{\alpha - i\beta}\right)$ —— (2)

Adding (1) and (2), we get

$$\Rightarrow 2C = \tan^{-1}\left(e^{\alpha+i\beta}\right) + \tan^{-1}\left(e^{\alpha-i\beta}\right)$$

$$\Rightarrow 2C = \tan^{-1}\left[\frac{e^{\alpha+i\beta} + e^{\alpha-i\beta}}{1 - e^{\alpha+i\beta}\cdot e^{\alpha-i\beta}}\right]$$

$$\Rightarrow 2C = \tan^{-1}\left\{\frac{e^{\alpha}\left(e^{i\beta} + e^{-i\beta}\right)}{1 - e^{2\alpha}}\right\}$$

$$\Rightarrow 2C = \tan^{-1}\left(\frac{e^{\alpha}\cdot 2\cos\beta}{1 - e^{2\alpha}}\right)$$

$$= \tan^{-1}\left(\frac{2\cos\beta}{e^{-\alpha} - e^{\alpha}}\right) = \tan^{-1}\left(\frac{2\cos\beta}{2\sinh\alpha}\right)$$

$$\Rightarrow 2C = \tan^{-1}\left(\frac{\cos\beta}{\sinh\alpha}\right)$$

$$\Rightarrow C = \frac{1}{2}\tan^{-1}\left(\frac{\cos\beta}{\sinh\alpha}\right)$$